

## Transportation Costs (or Trucker's Dilemma)

Math 1050 College Algebra Project

A truck driving 260 miles over a flat interstate at a constant rate of 50 miles per hour gets 7 miles to the gallon. Fuel costs \$3.50 per gallon. For each mile per hour increase in speed, the truck loses a tenth of a mile per gallon in its mileage. Drivers get \$27.50 per hour in wages and fixed costs for running the truck amount to \$11.33 per hour. What constant speed (between 50 mph and the speed limit of 65 mph) should the truck drive to minimize the total cost of the trip?

Part I: To solve a problem like this, it is a good idea to start with calculating an actual example. We will begin by finding the total cost if the truck is driven at 50 miles per hour. The cost is made up of two components: the fuel cost and the time cost. Be sure to include the correct units with each value below.

- A. Let's start out by finding how long the trip will take.

$$\frac{260}{50} = 5.2$$

The length of time required for the trip is 5.2 hrs. (Do not round.)

- B. Now, with this time known, how much will it cost to pay the driver and run the truck?

$$5.2(27.50 + 11.33)$$

$$5.2(38.83) = \$201.92$$

The amount needed to pay the driver and run the truck is \$201.92. (Round to nearest cent.)

- C. Next determine, at 7 miles per gallon for 260 miles, how much fuel will be required.

$$\frac{260}{7} = 37.143$$

The amount of fuel required is  $\frac{260}{7}$  gallons. (Do not round. Leave as a fraction.)

- D. With the amount of fuel known, how much will the fuel cost to make the 260 miles?

$$3.50\left(\frac{260}{7}\right) = 130$$

The cost of the fuel is \$130.00. (Round to nearest cent.)

- E. Finally we can find the TOTAL cost.

$$\begin{array}{r} 201.92 \\ + 130.00 \\ \hline 331.92 \end{array}$$

The total cost for the trip is \$331.92. (Round to nearest cent.)

Part II: The preceding process should have illuminated the basic procedure we will use to find the total cost of a trip. Next we will find the total cost if the truck is driven at 65 miles per hour. As in Part I, include the correct units with each value.

A. Let's find how long the trip will take.

$$\frac{260}{65} = 4$$

The length of time required for the trip is 4 hrs.. (Do not round.)

B. Now, with this time known, how much will it cost to pay the driver and run the truck?

$$4(38.83) = 155.32$$

The amount needed to pay the driver and run the truck is \$155.32. (Round to nearest cent.)

C. Next, to begin determining the fuel cost, we need to find the mileage (miles per gallon of fuel) when the truck is travelling at 65 miles per hour.

$$65 - 50 = 15$$

$$7 - \frac{1}{10}(15) = 5.5$$

The mileage at 65 miles per hour is 5.5 miles. (Do not round.)

D. With the fuel mileage known, how much fuel will be needed for the 260 miles?

$$\frac{260}{5.5} = 47.273$$

The amount of fuel required is  $\frac{260}{5.5}$ . (Do not round. Leave as a fraction.)

E. With the amount of fuel known, how much will the fuel cost to make the 260 miles?

$$3.5 \left( \frac{260}{5.5} \right) = 165.45$$

The cost of the fuel is \$165.45. (Round to nearest cent.)

F. Finally we can find the TOTAL cost.

$$\begin{array}{r} 155.32 \\ 165.45 \\ \hline 320.77 \end{array}$$

The total cost for the trip is \$320.77. (Round to nearest cent.)

Part III. We should now have a good process for determining the total cost of a trip for any rate of speed greater than or equal to 50 miles per hour. Next is to create a Total Cost function using X as the unknown rate in miles per hour. Simplify your answers and remember to include units. As you work through each step, test your answers by plugging in 50 mph and then 65 mph and comparing with results from parts I and II.

A. Let's find how long the trip will take.

The length of time required for the trip is  $\frac{260}{X}$ .

B. Now with this time known, how much will it cost to pay the driver and run the truck?

$$\frac{260}{X} \cdot \frac{38.83}{1} = \frac{10095.8}{X}$$

The amount of money needed to pay the driver and run the truck is  $\frac{10095.8}{X}$ .

C. Next, to begin determining the fuel cost, we need to find the mileage (miles per gallon of fuel) when the truck is travelling at X miles per hour.

$$\begin{aligned} 7 - \frac{1}{10}(x-50) \\ 7 - \frac{1}{10}x + 5 \\ 12 - \frac{1}{10}x \end{aligned}$$

The mileage at X miles per hour is  $12 - \frac{1}{10}x$ .

D. With the fuel mileage known, how much fuel will be needed for the 260 miles?

The amount of fuel required is  $\frac{260}{12 - \frac{1}{10}x}$ .

E. With the amount of fuel known, how much will the fuel cost to make the 260 miles?

$$3.5 \left( \frac{260}{12 - \frac{1}{10}x} \right) = \frac{910}{12 - \frac{1}{10}x}$$

The cost of the fuel is  $\frac{910}{12 - \frac{1}{10}x}$ .

F. Now we can find the TOTAL cost function. Express your function as C(X) =

TOTAL Trip Cost Function is  $C(X) = \frac{10095.8}{X} + \frac{910}{(12 - \frac{1}{10}x)}$

G. The last thing we should do is verify that this is the correct function by evaluating it at 50 mph and 65 mph to see if we get the same values we have previously computed.

$$C(50) = \frac{10095.8}{50} + \frac{910}{(12 - \frac{1}{10}(50))}$$

$$201.916 + 130$$

$$C(50) = \underline{\$331.92}$$

$$C(65) = \frac{10095.8}{65} + \frac{910}{(12 - \frac{1}{10}(65))}$$

$$155.32 + 165.45$$

$$C(65) = \underline{\$320.77}$$

Part IV. Assuming the function is modeling correctly, you need to calculate the minimum cost. Graph the Cost Function and find its minimum point. Sketch your graph here: Have the lower left point represent (50,315). You may use a graphing utility to help you find the minimum point.



The minimum COST for the trip is \$319.73

The MPH that minimizes COST is 61 mph

Reflective Writing: How did this project change the way you think about real-world math applications? Write at least one paragraph stating what ideas changed and why. Next, discuss how the math skills that you applied in this project will impact other classes you will take in your school career? Point to specific parts of the project and your own process in completing it that might have applications for other classes.